RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

SECOND YEAR
B.A./B.Sc. FOURTH SEMESTER (January – June) 2015
Mid-Semester Examination, March 2015

Date : 18/03/2015

MATHEMATICS (Honours)

Time : 11 am – 1 pm

Paper : IV

Full Marks : 50

[Use a separate answer book for each group]

<u>Group – A</u>

Select the correct alternative : [3×2]

1. Let X be dense in $\mathbb{R}$ and A be a proper subset of X. Then
   a) A and X – A both are dense in $\mathbb{R}$
   b) None of A and X – A is dense in $\mathbb{R}$
   c) A least one of A, X – A must be dense in $\mathbb{R}$
   d) None of these
2. Suppose A , B are subsets of a metric space (X, d) such that d(A, B) = 0. Then
   a) $A \cap B \neq \phi$
   b) There is a point in X which is common to both $\bar{A}$ and $\bar{B}$.
   c) There is a point in X which is a common limit point of A and B
   d) None of these
3. Consider the subset $\mathbb{Q}$ of $\mathbb{R}$ in usual metric
   a) No proper subset of $\mathbb{Q}$ is dense in $\mathbb{R}$
   b) Only countably many subsets of $\mathbb{Q}$ are dense in $\mathbb{R}$
   c) There are uncountably many subsets of $\mathbb{Q}$ which are dense in $\mathbb{R}$.
   d) None of these

4. **Answer <u>any two</u> :** [2×5]
   a) Define metric topology and equivalent metrics. Are any two metrics on a finite set equivalent? Justify your answer. [1+1+3]
   b) In a metric space, show that the union of two bounded sets is also bounded. If A is a subset of a metric space, show that diam (A) = diam ($\bar{A}$). [2+3]
   c) Define G_δ and F_σ sets in a metric space. Show that in a metric space, every singleton is G_δ.
   Find a subset A of $\mathbb{R}$ in usual metric such that A^d, the derived set of A is infinite and $A \cap A^d = \phi$. [3+2]

5. **Answer <u>any two</u> :** [2×3]
   a) For each $n \in \mathbb{N}$, let $f_n(x) = nx, \ 0 \le x \le \frac{1}{n}$
   $$= 1 \ , \ \frac{1}{n} < x \le 1$$
   Show that $\{f_n\}$ is not uniformly convergent on [0,1]
   b) For each $n \in \mathbb{N}$ let $f_n(x) = nx^2, 0 \le x \le \frac{1}{n}$
   $$= x \ , \ \frac{1}{n} < x \le 1$$
   Examine if $\{f_n\}$ is uniformly convergent on [0,1] or not.
   c) Let $f_n(x) = \frac{nx}{1+nx}, x \in [0,1]$. Show that $\{f_n\}$ is not uniformly convergent on [0,1].

6. State and prove Cauchy criteria for uniform convergence of the sequence of function $\{f_n\}$ on $D \subseteq \mathbb{R}$. [3]

Group – B

7. **Answer <u>any two</u> :** [2×5]

a) Obtain the pedal of $y^2 = 4ax$ with respect of vertex.

b) Find the pedal equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with respect to focus as pole.

c) Find the asymptates of the curve given by $y^4 - 2x^2y^2 - x^4 + 2axy^2 - 5ax^3 + 2x + 3y - 1 = 0$.

8. **Answer <u>any three</u> :** [3×5]

a) Solve : $\frac{dx}{x(y^2 - z^2)} = \frac{dy}{-y(z^2 + x^2)} = \frac{dz}{z(x^2 + y^2)}$

b) Obtain the necessary condition for integrability of the total differential equation Pdx + Qdy + Rdz = 0.

c) Solve $x^6 \frac{d^2y}{dx^2} + 3x^5 \frac{dy}{dx} + a^2y = \frac{1}{x^2}$, by changing the independent variable.

d) Find x, y as functions of t, where they satisfy the simultaneous equations $\frac{d^2y}{dt^2} - 16x = t$, $\frac{d^2x}{dt^2} - y = 1$ and y = 0, x = 0, $\frac{dx}{dt} = 1$, $\frac{dy}{dt} = -\frac{1}{4}$ at t = 0.

e) Find the eigen values λ_n and eigen functions $y_n(x)$ for the differential equation $\frac{d^2y}{dx^2} + \lambda y = 0$ satisfying the boundary conditions $y(0) = y(\pi)$ and $y'(0) = y'(\pi)$.

——————— × ———————